

# Combinatorics

B. Math. III

Final Examination

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**Instructions:** All questions carry ten marks. You may consult class notes and also van Lint-Wilson's book on Combinatorics during the examination, nothing else.

Duration of the exam is three hours.

1. Let  $D$  be a  $2 - (v, k, \lambda)$  design with  $r$  blocks through every point. Let  $B$  be any block of  $D$ . Prove that the number of blocks that meet  $B$  is at least

$$\frac{k(r-1)^2}{(k-1)(\lambda-1) + (r-1)}.$$

2. Prove that a binary code of length 6 and minimum distance 3 can have  $\leq 8$  codewords. Can the equality be attained? Justify
3. For a natural number  $n$ , let  $N(n)$  denote the number of mutually orthogonal Latin squares of order  $n$ . Given two natural numbers  $l$  and  $m$ , prove that

$$N(lm) \geq \min\{N(l), N(m)\}.$$

4. Let  $(M_{12})_B$  be the subgroup of the Mathieu group  $M_{12}$  that leaves a block  $B$  of the Witt design  $W_{12}$  invariant. Prove that  $(M_{12})_B$  acts as the full symmetric group  $S_6$  on the six elements of  $B$ .